Plane Wave Excitation of an Infinite Dielectric Rod

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Abstract—**The analysis of the fields induced inside and scattered externally by an infinite dielectric rod in an incident plane-polarized TEM wave is presented. It is assumed that the incident wave is polarized parallel to the rod, and that the dielectric has some significant loss. The analysis accounts for axial variation of the fields both inside and scattered by the rod. An investigation of the field distributions for a specific case is given, along with a brief discussion of the evaluation of integer order Bessel functions with complex arguement.**

I. Introduction

The PROBLEM of excitation of a dielectric rod has been widely studied [1], and it is well known that analyses of this kind are an important first step in considering problems such as the dielectric rod in a waveguide or cavity. Analysis problems involving scattering of fields from objects in free-space have long been in existence [2], yet to date there have been few published investigations of the field distributions induced inside the cylinder for the case where the object is electrically large and consists of a lossy dielectric material.

In many applications, such as microwave heating of dielectric objects, the exact field distribution within the object is of some interest. While investigations of this type are usually performed using numerical techniques such as finite domain/finite time (FD-TD) or finite element, the results produced by these methods are often limited to objects that are electrically small to moderate in size. The development of numerical methods such as FD-TD algorithms for examining microwave circuits and components has not diminished the need for analytically based models. While the geometries that can be considered by analytical-based models are necessarily simpler than those that can be handled by numerical techniques, an advantage is that analytically based models often offer insight into the individual contribution and effects of the various physical parameters of the problem. In some cases, analytical models are used to confirm the performance of numerical models in a similar manner to the way experimental results are often used to confirm the predictions of analytical models.

The method presented here for the analysis expresses the incident plane parallel wave as an infinite series of cylindrical waves whose origin lies at the center of the dielectric cylinder. Expressions for the fields scattered externally by the rod and total fields induced within the rod are given in terms of an infinite set of unknown cofficients to be determined by applying the appropriate boundary conditions The boundary conditions that the tangential components of the electric and magnetic fields are continuous across the dielectric boundary are applied in order to produce closed-form expressions for this set of unknown coefficients.

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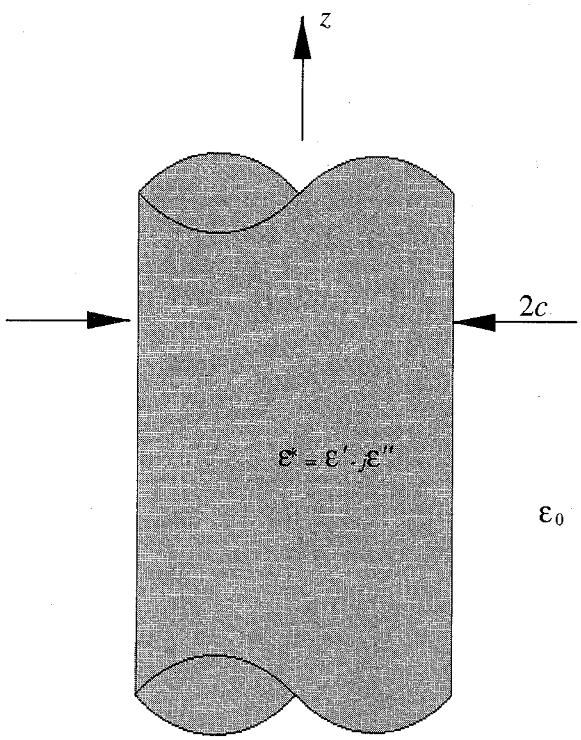

Fig. 1. Infinite dielectric rod in free space.

Results are presented that show the field scattered from and induced within a rod for both electrically small and large cases. It is shown that for cases of moderate dielectric permittivity that the fields may be assumed to be axially symmetric for values of rod diameter less than 10% of a wavelength.

II. Theory

Consider an infinitely long circularly cylindrical dielectric rod of radius $r = c$ and relative permittivity ϵ_r, which may be complex valued for dielectrics with loss, extending from $-\infty < z < \infty$ with center located at $x = 0$ and $y = 0$ (see Fig. 1). It is assumed that all field quantities have a time dependence of $e^{j\omega t}$ and that $\mu_r = 1$. The case examined here is that where there is a plane wave polarized with the electric field, E_z^{inc}, parallel to the z-axis incident on the rod.

By solving suitable boundary conditions at the rod surface, the scattered fields and field distribution inside the rod may be

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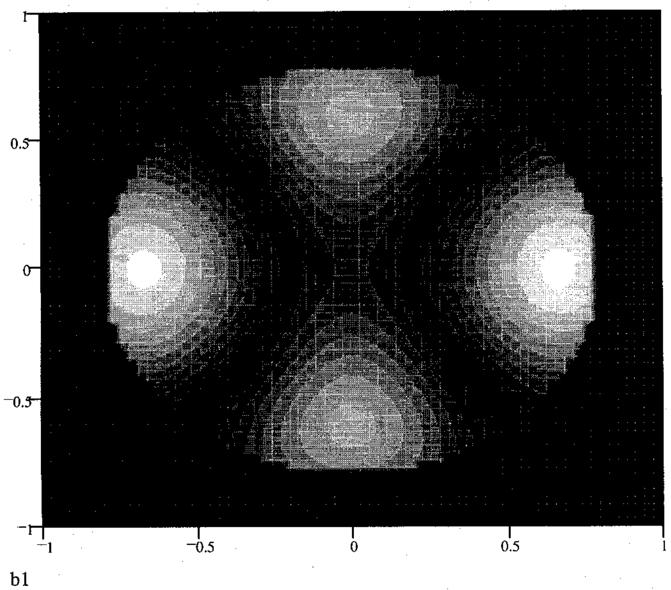

Fig. 2. Normalized electric field distribution induced inside a dielectric rod for the case where $f = 2$ GHz, $c = 20$ mm, and $\varepsilon_r = 20 - j1$.

determined. The incident plane wave may be written as:

$$E_z(x) = E_0 e^{-jkx} \tag{1}$$

where E_0 is the amplitude of the field and k is the wave number of the region external to the dielectric rod. This incident wave may be expressed in cylindrical co-ordinates as [3]:

$$E_z^{\text{inc}}(r,\theta) = E_0 \sum_{n=-\infty}^{\infty} (-j)^n J_n(kr) e^{jn\theta} \tag{2}$$

where J_n is an nth order Bessel junction of the first kind.

The associated magnetic field $H_\theta(r,\theta)$ may be found from Maxwell's equations [4], i.e.

$$\begin{aligned} H_\theta^{\text{inc}}(r,\theta) &= \frac{-j}{k\eta}\frac{\partial}{\partial r} E_z(r,\theta) \\ &= \frac{-jE_0}{k\eta} \sum_{n=-\infty}^{\infty} (-j)^n J_n'(kr) e^{jn\theta} \end{aligned} \tag{3}$$

where η is the intrinsic impedance of the region outside the dielectric rod (for free space $\eta = 377\ \Omega$) and J_n' is the first derivative of $J_n(z)$.

It can be shown that the fields scattered from the rod may be written as:

$$E_z^{\text{scat}}(r,\theta) = \sum_{n=-\infty}^{\infty} A_n H_n^{(2)}(kr) e^{jn\theta} \tag{4}$$

$$H_\theta^{\text{scat}}(r,\theta) = \frac{-j}{k\eta} \sum_{n=-\infty}^{\infty} A_n (H_n^{(2)})'(kr) e^{jn\theta} \tag{5}$$

where $H_n^{(2)}$ and $(H_n^{(2)})'$ is the nth order Hankel function of the second kind and first derivative. Similarly the fields inside the rod may by given by:

$$E_z^{\text{rod}}(r,\theta) = \sum_{n=-\infty}^{\infty} B_n J_n(k^d r) e^{jn\theta} \tag{6}$$

$$H_\theta^{\text{rod}}(r,\theta) = \frac{-j}{k\eta} \sum_{n=-\infty}^{\infty} B_n J_n'(k^d r) e^{jn\theta} \tag{7}$$

where $k^d = k\sqrt{\epsilon_r}$ is the wave number inside the dielectric rod [5], (a brief discussion of the evaluation of the first-order Bessel function for the case where the argument is complex is given in Appendix I) and where A_n and B_n are unknowns to be determined by applying the boundary condition that the tangential electric and magnetic fields are continuous across the dielectric boundary at $r = c$.

Applying the boundary conditions and noting that these conditions must be true for all values of θ, it can be shown that:

$$\begin{aligned} E_0(-j)^n J_n(kc) + A_n H_n^{(2)}(kc) &= B_n J_n(k^d c) \\ \frac{-jE_0}{k\eta}\left\{(-j)^n J_n'(kc) + A_n (H_n^{(2)})'(kc)\right\} &= \frac{-j}{k^d \eta^d} B_n J_n'(k^d c) \end{aligned} \tag{8}$$

From (8) it is straightforward to show that:

$$A_n = -E_0(-j)^n \frac{J_n'(kc) J_n(k^d c) - J_n'(k^d c) J_n(kc)}{J_n(k^d c)(H_n^{(2)})'(kc) - J_n'(k^d c) H_n^{(2)}(kc)} \tag{9}$$

$$B_n = E_0(-j)^n \frac{J_n(kc)(H_n^{(2)})'(kc) - J_n'(kc) H_n^{(2)}(kc)}{J_n(k^d c)(H_n^{(2)})'(kc) - J_n'(k^d c) H_n^{(2)}(kc)} \tag{10}$$

The electric field distribution scattered by the rod may be calculated by using (4) and (9). Similarly, the field distribution within the rod may by calculated using (6) and (10).

III. Result

The expressions derived in the previous section are now to be used to investigate the electric field distribution induced inside a rod for various values of rod diameter c and relative permittivity ϵ_r. It is assumed that the frequency of excitation is 2 GHz.

Fig. 2 shows the results (light regions refer to high field intensity) for the case $c = 20$ mm and $\epsilon_r = 20 - j1$ (i.e. $c/\lambda = 60\%$), and it can be seen that the fields show distinct regions of maxima and minima. This result has important consequences for the use of microwave energy in heating electrically large objects. Fig. 3 shows the results for the case where $c = 3.5$ mm and $\epsilon_r = 20 - j1$. For this case $c/\lambda = 10\%$, and it may be seen that the field distributions are approximately axially symmetric.

IV. Conclusion

A theoretical model of the fields induced in and scattered from an infinite dielectric rod by a plane-polarized TEM wave have been given. The analysis was performed by expressing the incident field as an infinite sum of cylindrical waves and solving the appropriate boundary conditions at the rod surface.

A brief investigation of the field distribution inside the rod for several specific values of rod diameter and relative permittivity showed that for a rod diameter less than 10% of a waveguide the fields inside the rod may be considered to be approximately axially symmetric.

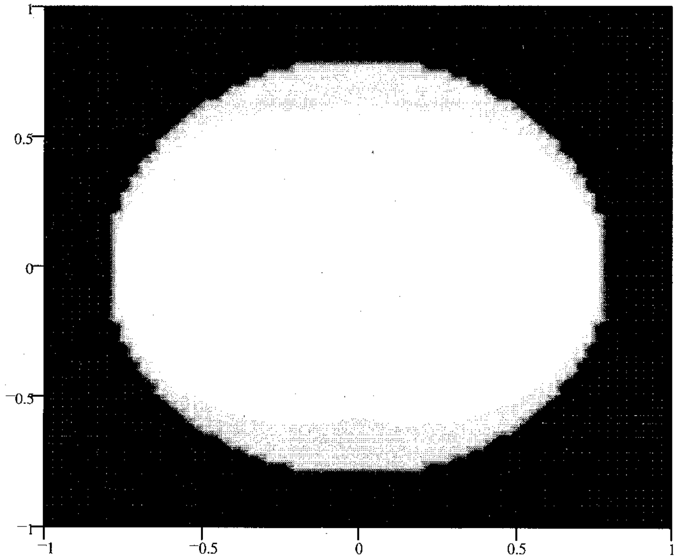

Fig. 3. Normalized electric field distribution induced inside a dielectric rod for the case where $f = 2$ GHz, $c = 3.5$ mm, and $\varepsilon_r = 20 - j1$.

APPENDIX

The results presented here require the evaluation of Bessel functions of the first and second kind for integer order and complex argument. While these functions are standard in many mathematical computer packages for either real or purely imaginary argument, often they are not for the case where the argument is complex.

Evalution of $J_n(z)$ is achieved using the algorithm [6] whereby the Fourier series coefficients of the Bessel generating function are determined:

$$J_n(z) = \frac{j^{-n}}{N} \sum_{k=0}^{N-1} e^{jz\cos(2\pi k/N)} e^{-j2\pi nk/N} \tag{11}$$

where N is related to the bandwidth of the generating function and should be large enough to comply with the Shanon-Kotelnikov sampling theorem [7]. The evaluation of $Y_n(z)$ is achieved using the following series expansion [3]:

$$\begin{aligned} Y_n(z) = {} & \frac{2}{\pi}\{\ln(x/2) + \gamma\} J_n(z) \\ & - \frac{1}{\pi} \sum_{k=0}^{n-1} (n-k-1)!(x/2)^{2k-n} \\ & - \frac{1}{\pi} \sum_{k=0}^{\infty} (-1)^k \{\Phi(k) + \Phi(n+k)\} \frac{(x/2)^{2k+n}}{k!(n+k)!} \end{aligned} \tag{12}$$

where $\gamma = 0.5772156$, and where $\Phi(p) = \sum_{k=1}^{p} 1/k$ and $\Phi(0) = 0$.

The Bessel function derivatives are calculated using the well known recurrence formulae:

$$B'_n(z) = \frac{nB_n(z)}{x} - B_{n+1}(z) \tag{13}$$

A complete discussion of evaluation of Bessel functions of the first and second kind for integer order and complex argument is given by du Toit [6].

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